$$\begin{bmatrix} \sigma_1^2 & & \\ \sigma_{12} & \sigma_2^2 & \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix}_{3\times 3} \qquad \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$$

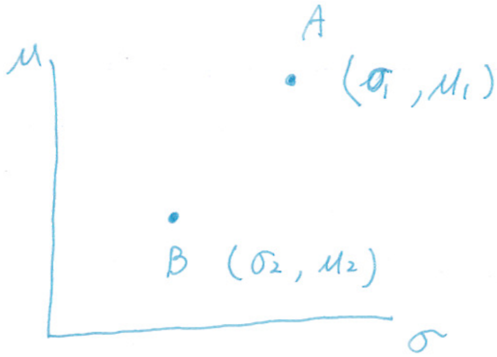
$n=2$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \qquad \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

座標

	Y μ	X σ
A	μ_1	σ_1
B	μ_2	σ_2

known



、兩個資產

X: A 資產的報酬率

Y: B 資產的報酬率

$E(X) = \mu_1 \quad Var(X) = \sigma_1^2 \qquad Cov(X,Y) = \sigma_{12}$

$E(Y) = \mu_2 \quad Var(Y) = \sigma_2^2$

$\sigma_{12} = \sigma_1 * \sigma_2 * \rho_{12}$

w_1: 投資在 X 資產的百分比 (權重)

w_2: Y

$$\begin{cases} w_1 + w_2 = 1 \\ w_1 \geq 0 \\ w_2 \geq 0 \end{cases} \qquad w_2 = 1 - w_1$$

$$r_p = \underbrace{w_1}_{\text{constant}} X + \overbrace{w_2}^{\text{r.v.}} Y = \text{r.v.}$$

Constant 常數

e.g.

Score = 0.3 * 期中考 + 0.4 * 期末考 + 0.3 * 作業

$= w_1 * X + w_2 * Y + w_3 * Z$

0.4   0.2   0.2

可以試算 兩科 x, y 之兩式 有不同~~組~~組合

$$E(r_p) = E[w_1 X + w_2 Y)$$

$$= E(w_1 X) + E(w_2 Y)$$

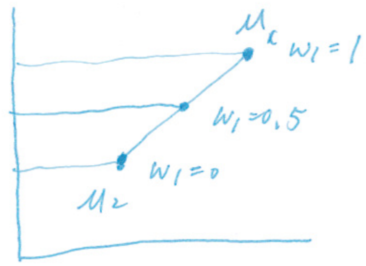
$$= \cancel{w_1 \mu_1 +}$$

$$= w_1 E(X) + w_2 E(Y)$$

$$= w_1 \mu_1 + w_2 \mu_2 \quad = w_1 \mu_1 + (1 - w_1)\mu_2$$

$$= w_1 \mu_1 + \mu_2 - w_1 \mu_2$$

$$= w_1(\mu_1 - \mu_2) + \mu_2$$



$$Var(r_p) = Var(w_1 X + w_2 Y)$$

$$= Var(w_1 X) + Var(w_2 Y) + 2\,Cov(w_1 X, w_2 Y)$$

$$(x+y)^2 = x^2 + y^2 + 2xy$$

$$\downarrow \quad \downarrow$$

乙方式有兩積 $\quad \sigma_1^2 + \sigma_2^2 + 2 * \sigma_{12}$

$$= w_1^2 Var(X) + w_2^2 Var(Y) + 2 w_1 w_2 \underline{Cov(X, Y)}$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

$$E(r_p) = w_1\mu_1 + w_2\mu_2 = w_1(\mu_1 - \mu_2) + \mu_2$$

$$Var(r_p) = \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\underline{w_1 w_2 \sigma_1 \sigma_2}\rho_{12}$$

$$(w_1\sigma_1)(w_2\sigma_2)\rho_{12}$$

$$-1 \le \rho_{12} \le 1$$

$$\boxed{\rho_{12} = 0}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2$$

$$\sigma_p = \sqrt{\quad \text{"} \quad}$$

$$\boxed{\rho_{12} = 1}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2(w_1\sigma_1)(w_2\sigma_2) * 1$$

$$= (w_1\sigma_1)^2 + (w_2\sigma_2)^2 + 2(w_1\sigma_1)(w_2\sigma_2)$$

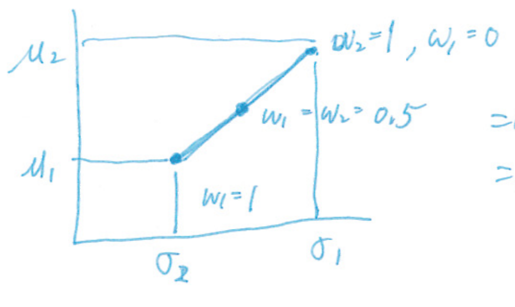
$$x^2 + y^2 + 2x * y = (x+y)^2$$

$$= (w_1\sigma + w_2\sigma_2)^2$$

$$\sigma_p = w_1\sigma_1 + w_2\sigma_2 = w_1\sigma_1 + (1-w_1)\sigma_2$$

$$= \cancel{(w_1 - w_2)\sigma}$$

$$= w_1(\sigma_1 - \sigma_2) + \sigma_2$$



$$= 0.5\sigma_1 - 0.5\sigma_2 + \sigma_2$$

$$= 0.5\sigma_1 + 0.5\sigma_2$$

$\boxed{\rho_{12} = -1}$

$$\sigma_p^2 = w_1^2 \sigma_2^2 + w_2 \sigma_2^2 + 2(w_1\sigma_1)(w_2\sigma_2) * (-1)$$

$$= (w_1\sigma_1)^2 + (w_2\sigma_2)^2 - 2(w_1\sigma_1)(w_2\sigma_2)$$

$$X^2 + Y^2 - 2(X * Y) = (X - Y)^2$$

$$= (w_1\sigma_1 - w_2\sigma_2)^2$$

$$\sigma_p = |w_1\sigma_1 - w_2\sigma_2|$$

$$= |w_1\sigma_1 - (1 - w_1)\sigma_2|$$

$$= |w_1\sigma_1 - \sigma_2 + w_1\sigma_2|$$

$$= |w_1\sigma_1 + w_1\sigma_2 - \sigma_2|$$

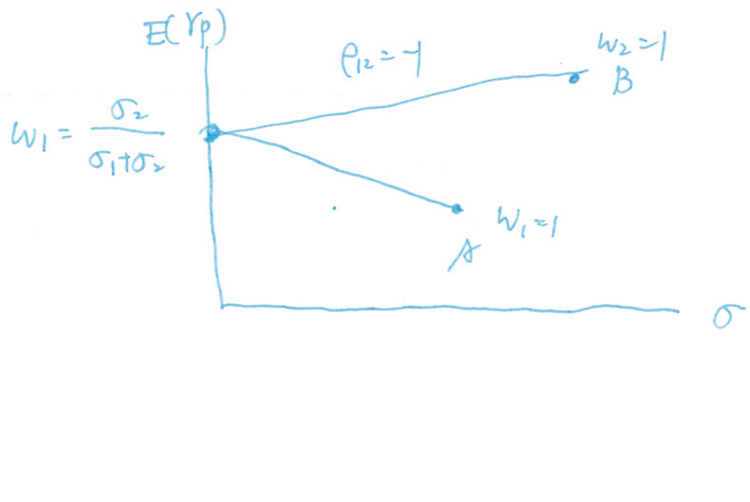
$$= |w_1(\sigma_1 + \sigma_2) - \sigma_2| \Rightarrow |w_1(\sigma_1+\sigma_2) - \sigma_2| = 0$$

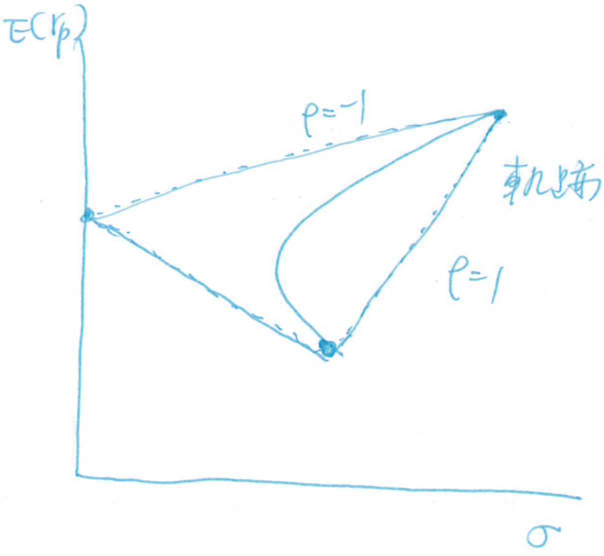
$$w_1(\sigma_1+\sigma_2) = \sigma_2$$

$$w_1 = \frac{\sigma_2}{\sigma_1+\sigma_2}$$

$$E(r_p) = w_1\mu_1 + w_2\mu$$

$$= w_1(\mu_1 - \mu_2) + \mu_2$$





$w_1 + w_2 = 1$

只要知道 w_1，w_2 就知道

$$
\begin{bmatrix} \sigma_1^2 & & \\ \sigma_{12} & \sigma_2^2 & \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix}
\quad
\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}
\quad
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}
$$

① $0 \le w_1 \le 1$

② $0 \le w_2 \le 1$

③ $0 \le w_3 \le 1$

④ $w_1 + w_2 + w_3 = 1$

$r_p = w_1 X + w_2 Y + w_3 Z$

	X	Y	Z
μ	μ_1	μ_2	μ_3
σ	σ_1^2	σ_2^2	σ_3^2

$E[r_p] = w_1\mu_1 + w_2\mu_2 + w_3\mu_3$ 線性

$Var(r_p) = Var(w_1 X + w_2 Y + w_3 Z)$

$$Var(w_1 X + w_2 Y + w_3 Z)$$

$=$

$$(X+Y+Z)^2$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2xz \quad \text{9項}$$

	w_1 X	w_2 Y	w_3 Z
$w_1 X$	$w_1^2 x^2$	$w_1 w_2$ xy	xz
$w_2 Y$	$w_1 w_2$ xy	y^2	$\boxed{yz}$
$w_3 Z$	$w_1 w_3$ xz	$\boxed{yz}$	$w_3 z^2$

對角線

$$1 = \rho_{11}, \quad 1 = \rho_{22}, \quad 1 = \rho_{33}$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2$$

$$+ 2 w_1 \sigma_1 w_2 \sigma_2 \rho_{12}$$

$$+ 2 w_1 \sigma_1 w_3 \sigma_3 \rho_{13}$$

$$+ 2 w_2 \sigma_2 w_3 \sigma_3 \rho_{23}$$

$$= \begin{array}{c|ccc} & X & Y & Z \\ \hline X & w_1^2\sigma_1^2 & & \\ Y & w_1\sigma_1 w_2\sigma_2 \rho & w_2^2\sigma_2^2 & \\ Z & w_1\sigma_1 w_3\sigma_3 \rho & & w_3^2\sigma_3^2 \end{array}$$

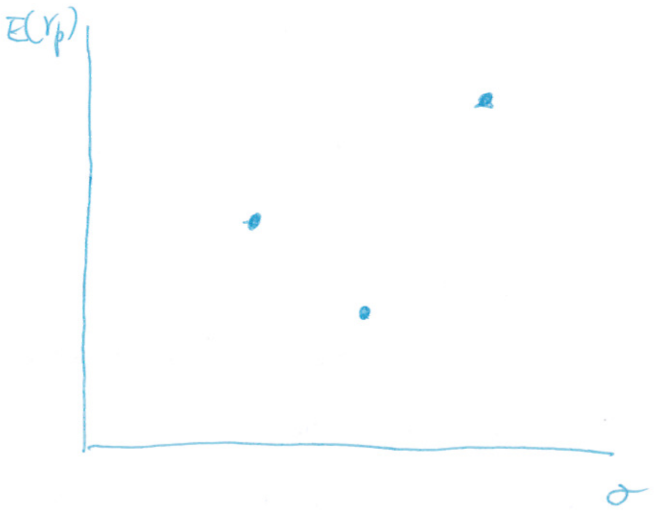
$$= \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_{12}, \sigma_{13}, \sigma_{23}$$
$$\rho_{12} \quad \rho_{13} \quad \rho_{23}$$

已知

$\mu_1 \quad \mu_2 \quad \mu_3$

未知是 w_1, w_2, w_3 的值 $w_3 = 1 - w_1 - w_2$

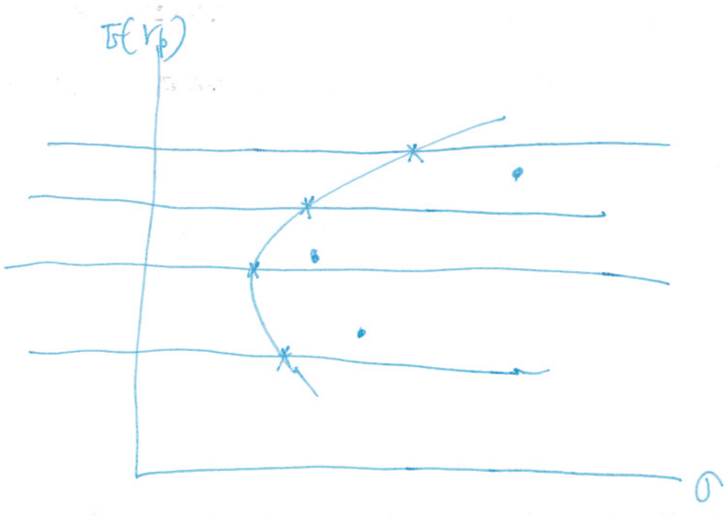
至少要知道兩個 才能決定軌跡



只要知道 $w_1, w_2 \Rightarrow w_3$ 就知道

则 $E(r_p) = w_1\mu_1 + w_2\mu_2 + (1-w_1-w_2)\mu_3$ 已知

$$Var(r_p) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + (1-w_1-w_2)^2\sigma_3^2$$
$$+ 2w_1\sigma_1 w_2\sigma_2\rho_{12}$$
$$+ 2w_1\sigma_1(1-w_1-w_2)\sigma_3\rho_{13}$$
$$+ 2w_2\sigma_2(1-w_1-w_2)\sigma_3\rho_{23}$$

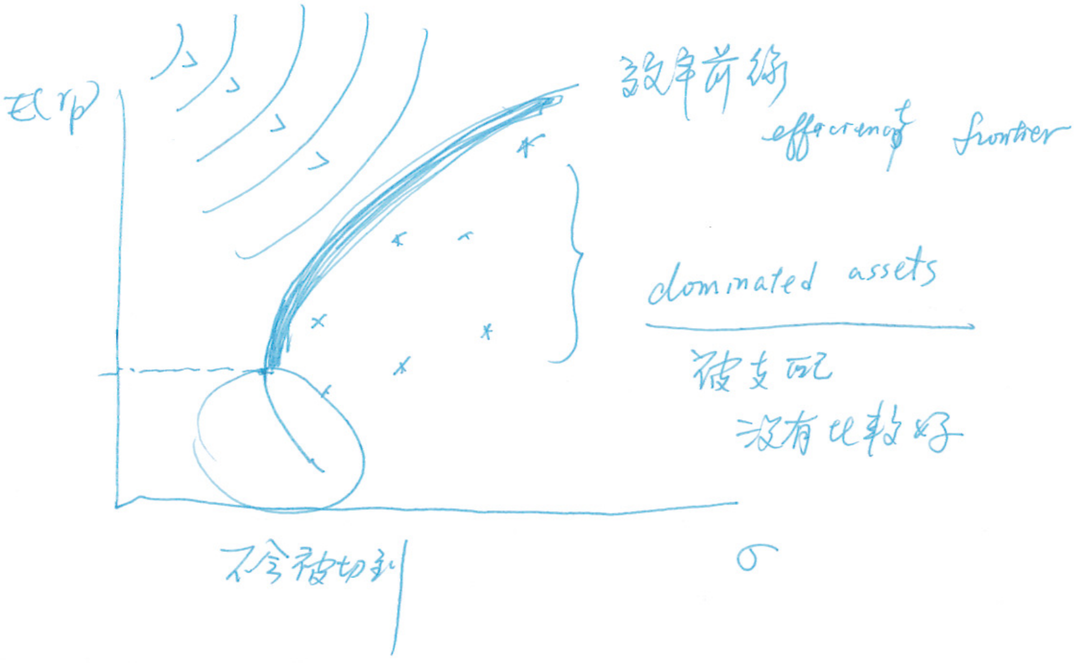


有個特徵 在固定的報酬率下 有最小的風險 的 $\boxed{w_1, w_2}$ 是否可以推得到

$$\min_{w_1, w_2} \sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1\sigma_1 w_2\sigma_2\rho_{12} \cdots\cdots}$$

s.t. [subject to]

$$w_1\mu_1 + w_2\mu_2 + w_3\mu_3 = C$$


E(rp)
效率前缘
efficiency frontier
dominated assets
被支配
没有比较好
不会被切到
σ