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Vermunt, J.K (2010). Mixture models for multilevel data sets. In: J. Hox and J.K. Roberts (eds.), Handbook of Advanced Multilevel Analysis, 59-81. New York: Routledge.

$$g[E(y_{ijk} \mid U_{jk}^{(2)}, U_k^{(3)}, X_{ijk}^{(1)}, Z_{ijk}^{(1)})] = \sum_{p=0}^{P} \beta_p x_{pijk}^{(1)} + \sum_{q=0}^{Q^{(2)}} \left(\sum_{c=1}^{C} \lambda_{qc}^{(1,2)} u_{jkc}^{(2)} \right) z_{qijk}^{(1,2)} + \sum_{q=0}^{Q^{(3)}} \left(\sum_{d=1}^{D} \lambda_{qd}^{(1,3)} u_{kd}^{(3)} \right) z_{qijk}^{(1,3)} \quad (6)$$

$$\text{logit}[P(u_{jkc}^{(2)} = 1 \mid U_k^{(3)}, X_{jk}^{(2)}, Z_{jk}^{(2)})] = \sum_{r=0}^{R} \gamma_{rc}^{(2)} x_{rjk}^{(2)} + \sum_{s=0}^{S} \left(\sum_{d=1}^{D} \lambda_{scd}^{(2,3)} u_{kd}^{(3)} \right) z_{sjk}^{(2,3)} \quad (7)$$

$$\text{logit}[P(u_{kd}^{(3)} = 1 \mid X_k^{(3)})] = \sum_{t=0}^{T} \gamma_{td}^{(3)} x_{tk}^{(3)} . \quad (8)$$

$$g[E(y_{ijk} \mid u_{jkc}^{(2)} = 1, u_{kd}^{(3)} = 1)] = \beta_0 + \beta_i + \lambda_{0c}^{(1,2)} + \lambda_{ic}^{(1,2)} + \lambda_{0d}^{(1,3)} + \lambda_{id}^{(1,3)} ,$$

$$\text{logit}[P(u_{jkc}^{(2)} = 1 \mid u_{kd}^{(3)} = 1)] = \gamma_{0c}^{(2)} + \lambda_{0cd}^{(2,3)} ,$$

$$\text{logit}[P(u_{kd}^{(3)} = 1)] = \gamma_{0d}^{(3)} .$$

```
variables
   groupid District_ID;
   caseid Case_ID;
   dependent Response binomial exposure=7;
   independent Year nominal, Religion nominal;
   latent U3 group nominal 2, U2 nominal 4;
equations
   Response <- 1 + Year + Religion + U2 + U2 Year
                  + U3 + U3 Religion;
   U2 <- 1;
   U3 <- 1;
```

Single response variable

L2 class=U2

L3 class=U3

U2=4

U3=2

L2 variable=year 4 categories

L3 variable=region 4 categories

Random intercepts + random slopes

Response <- 1+year +religion +U2 +U2 Year+U3 +U3 Religion;
1=Intercept
U2 = random intercepts at level 2
U3 = random intercepts at level 3
Year= effect coding, summation of coefficients=0
Religion=effect coding, summation of coefficients=0
U2 Year= random slopes at level 2
U3 religion=random slopes at level 3
U2 < -1;
U3 <-1;

Response <- 1+year +religion +U2 +U2 Year+U3 +U3 Religion+U2 U3;

U2 U3=interaction of random effects at level 2 and random effects at level3

Response <- 1+year +religion +U2 +U2 Year+U3 +U3 Religion;
U2 < -1+U3;
U3 <-1;

Response <- 1+year +religion +U2 +U2 Year+ U3 year +U3 +U3 Religion;
U3 Year= random slopes of year at level 3

more extended models are obtained by adding "+ `U3 Year`" to the model for the response variable, "+ `U3`" to the model for the level-2 classes, and "+ `U2 U3`" to the model for the response variable.

```
variables
   groupid family;
   dependent wordlist continuous, cards continuous, figures
      continuous, matrices continuous, animals continuous,
      occupations continuous;
   latent U3 group nominal 4, U2 nominal 3;
equations
   cards <- 1 + U2;
   figures <- 1 + U2;
   matrices <- 1 + U2;
   animals <- 1 + U2;
   occupations <- 1 + U2;
   wordlist;
   cards;
   figures;
   matrices;
   animals;
   occupations;
   U2 <- 1 + U3;
   U3 <- 1;
```

Single level/L1 level 6 continuous response variables

Level 2: 3 Classes U2

Level 3: 4 Classes U3

Cards <-1+U2;   Cards <-1|U2;??????

U2 <- 1+U3;

U3 <-1

There are 3 groups/classes at level 2.

Latent profile analysis

There are 4 groups at level 3.

U2 <-1+U3;

Conditional probabilities

Indirect effect U3 on responses

Response <- 1+ U2 +U3;

Direct effect U3 on responses

```
variables
   groupid ...
   dependent ...
   latent U3 nominal group 3, F1 continuous, F2 continuous;
equations
   wordlist <- (1) F1;
   cards <- 1 | U3 + F1 | U3;
   figures <- 1 | U3 + F1 | U3 + F2 | U3;
   matrices <- 1 | U3 + F1 | U3;
   animals <- (1) F2;
   occupations <- 1 | U3 + F2 | U3;
   wordlist - occupations;
   F1 <- 1 | U3;
   F2 <- 1 | U3;
   F1 | U3;
   F2 | U3;
   F1 <-> F2 | U3;
   U3 <- 1;
```

Level 1: 6 continuous

Level 2: 2 factors: F1 and F2

Level 3: 3 classes U3

Factor means ,

F1 <-1 |U3;

F2 <-1 |U3;

variances and covariances,

F1 |U3;

F2 |U3;

F1 <-> F2 |U3;

and loadings vary across U3

cards <- 1|U3+F1|U3;