上課筆記

10.1

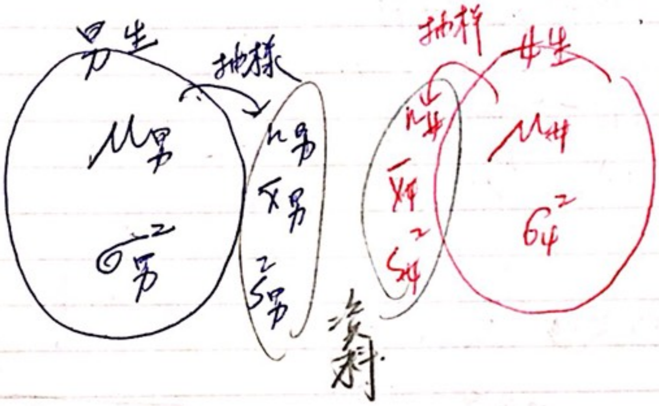
我想研究大学生畢業後第一份薪水，男女是否不一樣，怎麼做？

$H_0: \mu_{男} = \mu_{女}$

$H_1: \mu_{男} \neq \mu_{女}$

~~通常題目問的，會放在對立假設.~~

當研究放在對立假設



∵ $\sigma_{男}^2$ 與 $\sigma_{女}^2$ 是未知

∴ Z檢定不能使用

只能用t檢定（用s^2代替σ^2）

且要假設母体的薪資要服從常態分配.

（特別是樣本數不是很大時）

~~Step. H_0、H_1~~

先檢驗 $\sigma_{男}^2 - \sigma_{女}^2$ 是否成立→決定用哪個公式.

$H_0: \sigma_{男}^2 = \sigma_{女}^2$

$H_1: \sigma_{男}^2 \neq \sigma_{女}^2$

$\cancel{\sigma_{男}^2 - \sigma_{女}^2}$

$\frac{\sigma_{男}^2}{\sigma_{女}^2} = 1$ ⇒ 相等

假設 $\alpha = 0.05$

$s_{男}^2$　$s_{女}^2$ estimate of sigma(σ^2).

F_{v_1, v_2} 檢定 $= \frac{S_1^2}{S_2^2}$ (10.7, P356).

有二個自由度，大的 $v_1 = n_1 - 1$

小的 $v_2 = n_2 - 1$

$S_1^2 = \max(S_{男}^2, S_{女}^2)$

$S_2^2 = \min(S_{男}^2, S_{女}^2)$

假設 $\sigma_1^2 = \sigma_2^2$ 不被拒絕

Variance 相等 → pooled t.

不相等 → separate-variance t.

$$t = \frac{(\bar{X}_{男} - \bar{X}_{女}) - \cancel{(\mu_{男} - \mu_{女})}}{\sqrt{S_p^2\left(\frac{1}{n_{男}} + \frac{1}{n_{女}}\right)}}$$

→ 因為假設 $\mu_{男} = \mu_{女}$

pooled-variance t test

合併 combined.　$S_p^2 = S_{pooled}^2 = \hat{\sigma}_{男}^2 = \hat{\sigma}_{女}^2$　S_p^2 = weight-variance 加權變異數

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$$S_p^2 = \text{weighted-variance} = \frac{(n_{男}-1)S_{男}^2+(n_{女}-1)S_{女}^2}{(n_{男}-1)+(n_{女}-1)}$$

ex.10.2.

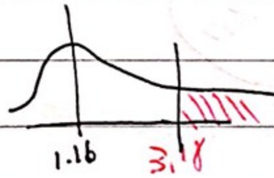
宣稱 Local 小於 Chain

$H_1: \mu_1 < \mu_2$. Local $n_1=10$ $\bar{x}=16.7$ $s^2=9.58$

$H_0: \mu_1 \geq \mu_2$ Chain $n_2=10$. $\bar{x}=18.88$ $s^2=8.22$

$$F = \frac{9.58}{8.22} = 1.166. \quad v_1=10-1=9, \quad v_2=10-1=9$$

Critical Value $= F_{v_1=9,\,v_2=9,\,\alpha=0.05}$ = 查表 ⇒ 3.18.



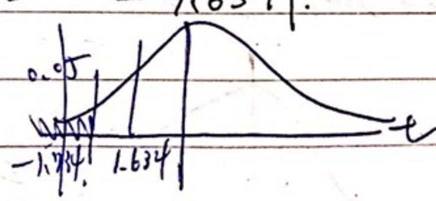
不拒絕 H_0. pooled-variance t test (assume $\sigma_1^2=\sigma_2^2$ 用10.7公式檢定) ($F=1.166$)

$$t = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{16.7-18.88}{\sqrt{8.8987\left(\frac{1}{10}+\frac{1}{10}\right)}} = -1.6341.$$

$$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{(n_1-1)+(n_2-1)}$$

$$= \frac{(10-1)9.58+(10-1)8.22}{(10-1)+(10-1)}$$

$= 8.8986$ (pooled variance).



若 $\sigma_1^2=\sigma_2^2$ is rejected

↓

separate-variance t 課本沒公式，但要調整自由度

$$t = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \qquad df = \frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}}$$

10.3 proportion

$H_0: \pi_1 = \pi_2$

$H_1: \pi_1 \neq \pi_2$

① ~~$\pi_1 = \pi_2$ 代表母體變異數~~

π是proportion ⇒ 二項分配.

二項分配的變異數來自π

如果比例π已知, 則變異數就已知

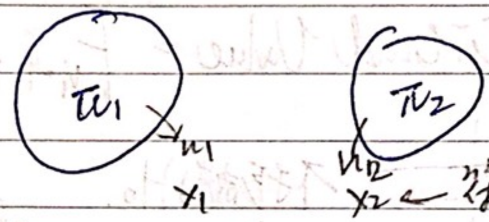
$\pi(1-\pi)$.

② ∵ $H_0: \pi_1 = \pi_2$

合併 pooled 不相等⇒即變異數相等.

$$z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} \leftarrow S_p^2$$

(課本10.5).



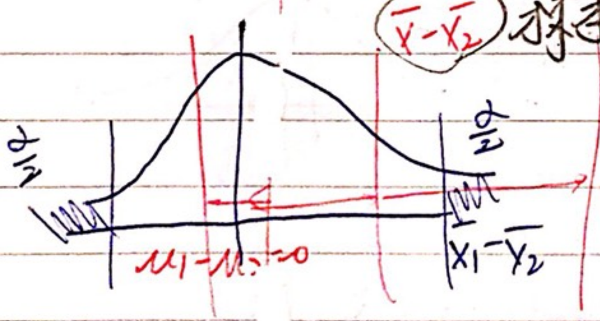
估計值 母體 $\hat{\pi}_1 = p_1$ 樣本統計量 $= \frac{x_1}{n_1}$ 因為 $H_0: \pi_1 = \pi_2$

$\hat{\pi}_2 = p_2 = \frac{x_2}{n_2}$ $\quad \bar{p} = \hat{\pi} = \hat{\pi}_1 = \hat{\pi}_2$

$= \frac{x_1 + x_2}{n_1 + n_2}$

$H_0: \mu_1 = \mu_2$

$\bar{x}_1 - \bar{x}_2$ 樣本的結果落在何處.



信賴區間 ⇒ 以 (抽樣) $\bar{x}_1 - \bar{x}_2$ 為中心看有無包含0

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20180322 Business Statistics

上課練習題第一題

18 subjects rated two brands of city coffee in a taste-testing experiment. A rating on 7-point scale (1=extremely unpleasing, 7=extremely ping) is given for each of four characteristics: at the 0.05 level of significance, is there evidence of a different in the mean rating between the two brands?

Brand A	Brand B
24	26
27	27
19	22
24	27
22	25
26	27
27	26
25	27
22	23

step1 $H_0 = \mu_A = \mu_B$

$H_A: \mu_A \neq \mu_B$.

step2 $\alpha = 0.05$.

Critical value. $= \pm 1.96$.

$n_A = 9$ $n_B = 9$ $S_A^2 = 7$ $S_B^2 = 3.5$ $X_A = 24$ $X_B = 25.55$

step3. $F = \frac{7}{3.5} = 2$

$F_{v_1 = 8\ v_2 = 8,\ \alpha = 0.5}$ 查表 $\rightarrow 4.43$.

$\bar{X}_A = 24$ $\bar{X}_B = 25.55$

$n = 9$.

$S_A^2 = [(27-24)^2 + (19-24)^2 + (22-24)^2 + (26-24)^2 + (27-24)^2 + (25-24)^2 + (22-24)^2] / (9-1)$

$= (9 + 25 + 4 + 4 + 9 + 1 + 4) / 8 = 7$

$S_A = \sqrt{7}$

$S_B^2 = [(26-25.55)^2 + (27-25.55)^2 + (22-25.55)^2 + (27-25.55)^2 + (25-25.55)^2 + (27-25.55)^2 + (26-25.55)^2 + (27-25.55)^2 + (23-25.55)^2] / 8$

$= (0.202 + 2.102 + 12.602 + 2.102 + 0.302 + 2.102 + 0.202 + 2.102 + 6.502) / 8$

$= 3.53$

$S_B = \sqrt{3.53}$

step4 $t = \frac{24 - 25.55}{\sqrt{5.25 \left(\frac{2}{9}\right)}}$

$S_p^2 = \frac{(9-1)7 + (9-1)3.5}{16}$

$= \frac{56 + 28}{16}$

$= 5.25$

$= \frac{-1.55}{\sqrt{1.166}}$

$= \frac{-1.55}{1.080}$

$= -1.435$

$t_{16, 0.025} = 2.1199$.

∴ 沒有證據証明其mean不同。 不拒絕

-2.1199

② 改成 9人(相關樣本.10.2).

上課練習題第一題

f test

Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	9
Sample Variance	7
Smaller-Variance Sample	
Sample Size	9
Sample Variance	3.5277
Intermediate Calculations	
F Test Statistic	1.9843
Population 1 Sample Degrees of Freedom	8
Population 2 Sample Degrees of Freedom	8
Two-Tail Test	
Upper Critical Value	4.4333
p-Value	0.3520
Do not reject the null hypothesis	

Calculations Area	
F.DIST.RT value	0.1760

1. p值=0.3520大於α = 0.05

無法拒絕兩品牌平均數相等的假設

上課練習題第一題

	A	B	C	D
	合併T			
	(assumes equal population variances)			
	Data			
	Hypothesized Difference	**0**		
	Level of Significance	**0.05**		
	Population 1 Sample			
	Sample Size	**9**		
	Sample Mean	**24**		
	Sample Standard Deviation	**2.6457513**		
	Population 2 Sample			
	Sample Size	**9**		
	Sample Mean	**25.5555**		
	Sample Standard Deviation	**1.878237**		
	Intermediate Calculations			
	Population 1 Sample Degrees of Freedom	8		
	Population 2 Sample Degrees of Freedom	8		
	Total Degrees of Freedom	16		
	Pooled Variance	5.2639		
	Standard Error	1.0816		
	Difference in Sample Means	-1.5555		
	***t* Test Statistic**	**-1.4382**		
	Two-Tail Test			
	Lower Critical Value	**-2.1199**		
	Upper Critical Value	**2.1199**		
	***p*-Value**	**0.1696**		
	Do not reject the null hypothesis			

1. p值=0.1696大於α = 0.05

無法拒絕兩品牌平均數相等的假設

人.	A	B		$D_i-\bar{D}$	$(D_i-\bar{D})^2$
1	24	26	-2	-0.44	0.1936
2	27	27	0	1.56	2.4336
3	19	22	-3	-1.44	2.0736
4	24	27	-3	-1.44	2.0736
5	22	25	-3	-1.44	2.0736
6	26	27	-1	0.56	0.3136
7	27	26	1	2.56	6.5536
8	25	27	-2	-0.44	0.1936
9	22	23	1	0.56	0.3136
			-14		16.2324

$\frac{-14}{9} = -1.56 = \bar{D}$

回家作業題

Step 1 $H_0: \mu_D = 0$

$H_A: \mu_D \neq 0$

$\alpha = 0.05$

$n = 9$

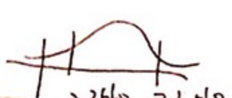
Step 2

$$t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$$

$$S_D = \sqrt{\frac{16.2324}{8}} = 1.4244$$

$$t = \frac{-1.56}{\frac{1.4244}{\sqrt{9}}} = -3.286$$

Step 3 $t_{0.025,8} = \pm 2.3060$



-3.281 → 拒絕H_0

∴ 有証据推翻 $\mu_A = \mu_B$

回家作業題

Paired t	
Data	
Hypothesized Mean Difference	**0**
Level of significance	**0.05**
Intermediate Calculations	
Sample Size	9
DBar	-1.5556
Degrees of Freedom	8
S_D	1.4240
Standard Error	0.4747
***t* Test Statistic**	**-3.2772**
Two-Tail Test	
Lower Critical Value	**-2.3060**
Upper Critical Value	**2.3060**
***p*-Value**	**0.0112**
Reject the null hypothesis	

1.因為-3.2772小於-2.3060
可以拒絕兩品牌平均數相等的假設
2. p值=0.0112小於α = 0.05
可以拒絕兩品牌平均數相等的假設